

# Preface

These notes were assembled during the spring 2026 semester of the second-year PhD macroeconomics sequence at Penn State, taught by Maria-Jose Carreras-Valle (Part I) and Kai-Jie Wu (Part II). They aim to serve simultaneously as a compact reference for the technical machinery of modern macroeconomics—heterogeneous-agent equilibria, dynamic programming, business-cycle accounting, the empirics of consumption—and as a self-contained narrative of how the field’s central questions evolve from one chapter to the next.

## Audience and Prerequisites

The intended reader is a first- or second-year graduate student who has had a careful undergraduate or master’s-level treatment of microeconomic theory (consumer choice, general equilibrium, basic dynamic programming) and the standard probability and real-analysis tools that come with that. No prior macroeconomics is strictly required, but the pace of *Part I* assumes familiarity with the Arrow–Debreu framework and the language of state-contingent claims.

## Structure of the Book

The book is divided into two parts, reflecting the two-instructor structure of the course.

**Part I: Heterogeneous Agents in Complete and Incomplete Markets** (Chapters 1–3, by Maria-Jose Carreras-Valle) develops a unified framework for studying risk sharing across heterogeneous agents. Chapter 1 establishes the complete-markets benchmark—Arrow–Debreu trading, sequential trading, the recursive social planner—against which the rest of the book pushes. Chapter 2 introduces *exogenous* market incompleteness through Huggett, Aiyagari, and Krusell–Smith. Chapter 3 turns to *endogenous* incompleteness arising from participation frictions: one-sided lack of commitment, the Bulow–Rogoff model, and two-sided lack of commitment. The three chapters share a methodological signature: equilibria are characterized by the cross-sectional distribution of state variables, and the natural recursive formulation uses promised utility (or its analogue) as the state.

**Part II: Growth, Business Cycles, and Quantitative Macroeconomics** (Chapters 4–11, by Kai-Jie Wu) takes the dynamic-equilibrium machinery and applies it to canonical macroeconomic questions. Chapter 4 develops growth and development accounting as the empirical hook. Chapters 5–7 build the Solow and neoclassical growth models and confront them with cross-country convergence data. Chapter 8 extends to Real Business Cycles, and Chapter 9 inverts the RBC model to perform Business Cycle Accounting. Chapter 10

treats consumption and saving theory—the Permanent Income Hypothesis, Hall’s Random Walk Hypothesis, and the empirical literature documenting excess sensitivity. Chapter 11 closes with the computation of the Aiyagari heterogeneous-agent model, which serves as the bridge into the modern HANK literature.

## Pedagogical Conventions

Several typographic conventions recur throughout the text.

- **Definitions** appear in green-shaded boxes. **Theorems, Propositions, Lemmas, Corollaries**, and **Claims** appear in cyan-shaded boxes; their proofs follow inline (or in a dedicated grey-bordered block, when emphasized).
- **Remarks** come in two flavors. The shorter *inline* remarks (`\rmk`) flag a brief point in the surrounding narrative; the boxed *block* remarks (`\rmkb`) develop a substantial side topic, often spanning several paragraphs and including subsidiary figures or tables.
- **Algorithms** (e.g. Value Function Iteration, Aiyagari’s outer loop) appear in violet-shaded boxes, listing the steps in order with implementation notes.
- **Examples** appear in their own environment with the worked solution clearly demarcated.
- **Facts** report empirical regularities in their own boxes, typically appearing in chapters that confront theory with data.

Each chapter opens with a brief *Notation in This Chapter* table listing chapter-specific symbols. The book-wide *Notation* section (immediately following this preface) collects symbols common to multiple chapters.

## Reading Paths

Readers do not have to proceed linearly.

- *Heterogeneous-agent macro focus.* Read Part I in full, then Chapter 11 (Aiyagari computation). Chapter 10’s PIH section provides useful background for the household problem in Aiyagari but is not strictly required.
- *Growth focus.* Read Chapters 4–7 as a self-contained block on growth theory and its cross-country evidence.
- *Business cycles focus.* Chapters 8–9 are the core; Chapter 10’s RWH section complements the empirical discussion.
- *Computational focus.* Chapter 6 (Section on VFI), Chapter 8 (RBC numerical solution), and Chapter 11 (Aiyagari) form a sequence of progressively harder computational exercises.

## Acknowledgments

These notes would not exist without Maria-Jose Carreras-Valle and Kai-Jie Wu, whose lectures form the underlying material. Any errors are mine—both as the typesetter and as the student.

Rui Zhou, Spring 2026

# Notation

The following symbols recur throughout the notes. Where a chapter departs from a convention listed here, a chapter-specific note is provided in its opening section. A few high-level conventions:

- **Lowercase vs. uppercase letters.** Lowercase letters (e.g.  $c, k, y$ ) denote per-worker or per-capita quantities. Uppercase letters (e.g.  $C, K, Y$ ) denote aggregates. The convention is occasionally relaxed in specific chapters; when it matters, the chapter's notation note flags the exception.
- **Time subscripts.**  $t$  indexes the period;  $T$  is the terminal period in finite-horizon problems and the simulation length in numerical sections.
- **States and histories.**  $s_t \in S$  is the period- $t$  exogenous state;  $s^t = (s_0, s_1, \dots, s_t)$  is the history through date  $t$ .
- **Conditional expectation.**  $\mathbb{E}_t[\cdot]$  denotes expectation conditional on the time- $t$  information set.

## Symbols used throughout the book.

Symbol	Meaning
<i>Preferences and discounting</i>	
$u(\cdot)$	Period utility function; $u' > 0$ , $u'' < 0$ , satisfying Inada conditions where needed.
$\beta$	Time discount factor; $\beta \in (0, 1)$ .
$\sigma$	Coefficient of relative risk aversion under CRRA utility; the inverse $1/\sigma$ is the intertemporal elasticity of substitution.
$\gamma$	Coefficient of <i>absolute</i> risk aversion under CARA utility (Ch. 2 only).
$\mathbb{E}_t[\cdot]$	Expectation conditional on history $s^t$ .
<i>Stochastic environment</i>	
$s_t, s^t$	Date- $t$ state; history through $t$ .
$\pi(s^t)$	Unconditional probability of history $s^t$ ; $\pi(s^\tau   s^t)$ is conditional.
$\varepsilon_t$	Innovation / shock realization.
$\rho$	Persistence parameter of an AR(1) process; $\rho = \psi$ in Ch. 2's CARA example.
<i>Endowment and production</i>	
$y(s^t), Y_t$	Stochastic endowment; aggregate output.

(continued on next page)

Symbol	Meaning
$F(K, L)$	Aggregate production function, typically constant returns to scale.
$f(k)$	Per-worker production function $f(k) = F(k, 1)$ .
$A, a_t$	Total factor productivity (TFP); $a_t = \ln A_t$ for the log-linear AR(1) version.
$\alpha$	Capital share in Cobb–Douglas production; output elasticity of capital.
$\delta$	Depreciation rate of physical capital; $\delta \in (0, 1]$ .
<i>Quantities</i>	
$c, C$	Consumption (per worker / aggregate).
$k, K$	Physical capital (per worker / aggregate).
$L, l$	Labor (aggregate / per worker). $L = 1$ in many setups.
$I_t$	Aggregate investment, $I_t = K_{t+1} - (1 - \delta)K_t$ .
$a, A$	Asset / debt holdings (note: $A$ is also used for TFP and natural debt limit; context disambiguates).
<i>Prices and returns</i>	
$r$	Real interest rate. Convention varies: in Ch. 1–3, 5–10, $r$ is the net rate or rental rate of capital; in Ch. 11, $r = F_K(K, L)$ is the rental rate and the household’s gross return is $1 + r - \delta$ . Each chapter’s notation note specifies the convention used.
$R$	Gross interest rate; typically $R = 1 + r$ .
$w$	Real wage.
$q(s^t)$	Date-0 Arrow–Debreu price of a state-contingent claim (Ch. 1).
$Q(s^t s)$	One-period-ahead pricing kernel in sequential trading (Ch. 1, 2).
<i>Solution objects</i>	
$V$	Value function.
$g(\cdot)$	Policy function.
$\Lambda, \lambda$	Cross-sectional distribution of agents (Ch. 2, 11).
<i>Lagrangian and shadow prices</i>	
$\mathcal{L}$	Lagrangian.
$\lambda^i, \mu^i$	Pareto weight or Lagrange multiplier on a specific agent’s budget; context distinguishes from the distribution $\lambda$ .
$\theta(s^t)$	Multiplier on resource constraint (planner’s problem, Ch. 1).
<i>Empirical / decomposition objects</i>	
Var, Cov	Cross-sectional variance and covariance.
$g_x$	Average growth rate of variable $x$ over a sample period (Ch. 4).

A few overloaded symbols deserve attention. The Greek letter  $\lambda$  is used both for Pareto weights / Lagrange multipliers and for the cross-sectional distribution of agents—the role is always clear from context. The letter  $A$  is used for both the natural debt limit (Ch. 1) and TFP (Ch. 5 onward); these never appear together. The letter  $a$  is used for asset holdings throughout, and as log-TFP in Ch. 8; again no overlap.

Each chapter opens with a brief notation note flagging any chapter-specific symbols and confirming the local interpretation of  $r$  and a few other context-dependent objects.

## Part I

# Heterogeneous Agents in Complete and Incomplete Markets

*Lectures by Maria-Jose Carreras-Valle*

## Part II

# Growth, Business Cycles, and Quantitative Macroeconomics

*Lectures by Kai-Jie Wu*

## Chapter 5

# The Solow Growth Model

Remark (Notation in This Chapter).

Symbol	Meaning
$A, A_t$	TFP, fixed in baseline; $A_t = A_0 e^{g_A t}$ in the BGP extension
$Y_t, y_t$	Aggregate / per-worker output
$K_t, k_t$	Aggregate / per-worker capital
$L$	Labor force, fixed in the baseline
$s$	Saving rate (exogenous, constant)
$\delta$	Depreciation rate
$g_A, g_k, g_y$	Steady-state growth rates of TFP, capital, output (BGP)
$k^*$	Steady-state per-worker capital
$k_0^*$	Initial capital stock placing the economy on the BGP

The Solow model is the fundamental workhorse of economic growth theory, focusing on how **capital accumulation** drives output growth over time. Its economic intuition rests on two core properties of physical capital:

- **Rivalry and Diminishing Returns:** Capital is a rival good. As we equip a fixed number of workers with more and more machines, the marginal contribution of each additional machine declines.<sup>1</sup>
- **Durability and Depreciation:** Capital is durable, allowing it to be accumulated over time, which generates a dynamic growth path. However, it also depreciates, meaning continuous investment is required to maintain the stock.

Key Takeaways from the Solow Model:

<sup>1</sup>To build intuition for why rivalry causes diminishing returns, consider the “congestion” of physical tools. If a single worker is given one unit of capital (e.g., a machine), there is no conflict. If a second unit is added, the two machines must now compete (or “conflict”) for the same worker’s fixed physical attention. This congestion dictates that the second machine cannot contribute as much as the first. A third machine creates even more conflict, yielding an even smaller marginal gain.

In contrast, a non-rival factor (like a software algorithm, a blueprint, or a scientific formula) does not suffer from this physical congestion. A single idea can be applied to all machines and workers simultaneously without being depleted. Therefore, an improvement in a non-rival factor acts as a direct multiplier to overall output, avoiding the trap of diminishing marginal returns.

- **Long-Run Stagnation:** Due to diminishing returns to capital, growth driven solely by capital accumulation will eventually slow down and cease.
- **Transitional Dynamics:** Economies that start with a low initial capital stock will experience strong “catch-up growth” as they converge to their steady state.

## 5.1 The Economic Environment

- **Time:** Discrete time,  $t = 0, 1, 2, \dots$ .
- **Production Function:** In each period, the total output  $Y_t$  is produced using physical capital  $K_t$  and labor  $L$ :

$$Y_t = AF(K_t, L)$$

where the technology level  $A$  is exogenously given and fixed.

### Remark.

In reality, output depends on total *effective labor* (namely the human capital,  $H$ ), which incorporates workers’ education and skills (i.e.,  $H = h \cdot L$ , where  $h$  is human capital per worker). However, in the baseline Solow model, we deliberately make two independent simplifying assumptions to isolate the mechanics of physical capital ( $K$ ):

1. We abstract away from skill accumulation by assuming all workers are identical and have a constant baseline skill level ( $h = 1$ ). Thus, human capital reduces to raw headcount:  $H = L$ .
2. We abstract away from population dynamics by assuming the raw labor force  $L$  is fixed and given (i.e., population growth rate  $n = 0$ ).

By shutting down the channels from both the education and the population, we force physical capital accumulation to be the sole driver of transitional growth in this baseline model.

- **Diminishing Returns to Capital:** The production function  $F(\cdot)$  is strictly concave in capital, meaning that the marginal product of capital decreases as we accumulate more capital. Formally, we assume  $F_K > 0$  and  $F_{KK} < 0$ .

In addition, we assume the production function  $F(\cdot)$  exhibits *Constant Returns to Scale (CRS)*. This allows us to divide both sides by  $L$ :

$$y_t = AF\left(\frac{K_t}{L}, 1\right) \equiv Af(k_t)$$

where  $f'(k_t) > 0$  and  $f''(k_t) < 0$ , capturing the strictly positive but diminishing marginal product of capital.

- **Law of Motion for Capital:** The evolution of the aggregate capital stock is governed by:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where  $\delta \in (0, 1)$  is the constant depreciation rate, and  $I_t$  is aggregate investment.

Intuitively, the future stock equals the current stock minus depreciation (outflow) plus new investment (inflow).

**Remark.**

If we assume that the depreciation rate increases with the capital stock (rather than being a constant  $\delta$ ), it would amplify the effect of diminishing returns, making the growth path of capital even more concave and slowing down convergence further.

- **Saving Behavior:** A constant fraction of total output is saved and invested.

$$I_t = sY_t$$

where  $s \in (0, 1)$  is the exogenous saving rate.

**Remark (Constant Saving Behavior).**

The assumption that saving is a constant fraction of output ( $I_t = sY_t$ ) is a purely mechanical behavioral rule, lacking strict microfoundations. It implies that households are myopic—they do not adjust their saving behavior in response to changes in the interest rate (the marginal product of capital) or expectations about future technological progress.

Despite its simplicity, this assumption is still justified in macroeconomics:

1. **Empirical Validity:** It aligns with Kaldor’s stylized facts, which observe that the aggregate investment-to-GDP ratio remains remarkably stable over long historical periods.
2. **Theoretical Robustness as a Limiting Case:** In more advanced frameworks (such as the Ramsey-Cass-Koopmans model), under some conditions, the optimal, fully forward-looking saving path actually collapses exactly to a constant saving rate.

Therefore, the Solow model’s assumption serves as a highly tractable, reduced-form approximation that successfully captures the core qualitative dynamics of capital accumulation without the heavy mathematical machinery of dynamic optimization.

Our goal is to characterize the dynamic path of the economy in per-worker terms:

$$\{y_t, k_t\}_{t=0}^{\infty}.$$

## 5.2 Solving the Model

To trace the growth path, we combine the law of motion with the saving assumption:

$$K_{t+1} = (1 - \delta)K_t + sY_t$$

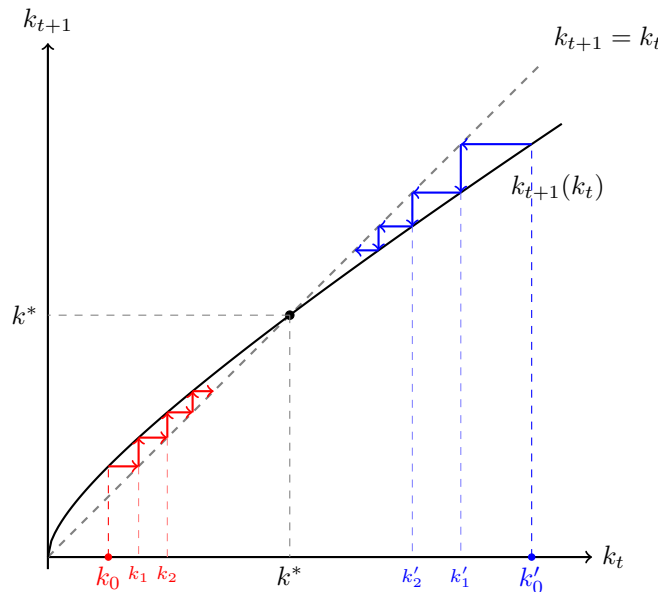
Dividing both sides by the constant labor force  $L$ , we obtain the fundamental difference equation of the Solow model in per-worker terms:

$$k_{t+1} = (1 - \delta)k_t + sy_t = (1 - \delta)k_t + sAf(k_t)$$

Because the production function  $f(k_t)$  exhibits diminishing marginal returns, the transition mapping  $k_{t+1}$  as a function of  $k_t$  is strictly concave.

Given any initial capital endowment  $k_0$ , we can solve for the entire dynamic path  $\{k_t\}_{t=0}^{\infty}$  by iterating on this law of motion. The intersection of this concave transition curve with the 45-degree line determines the unique steady state  $k^*$ , where the economy stops growing.

The phase diagram below visually traces out this dynamic growth path of  $k$  over time, illustrating how an economy starting from an initial  $k_0 < k^*$  accumulates capital and converges to the steady state.



Based on the phase diagram and the mechanics of capital accumulation, we can derive two fundamental implications of the baseline Solow model:

- **No Long-Run Growth:** In the absence of continuous technological progress, the economy inevitably converges to a fixed steady state ( $k^*$ ). Once there, the growth rate of capital and output per worker strictly drops to zero. This implies that capital accumulation alone cannot act as the engine of sustained long-run economic growth.
- **Catch-Up Growth (Transitional Dynamics):** Economies starting further below their steady state will grow at a faster rate. The underlying mechanism is strictly driven by the *diminishing marginal returns to capital*. When capital is scarce, its marginal product

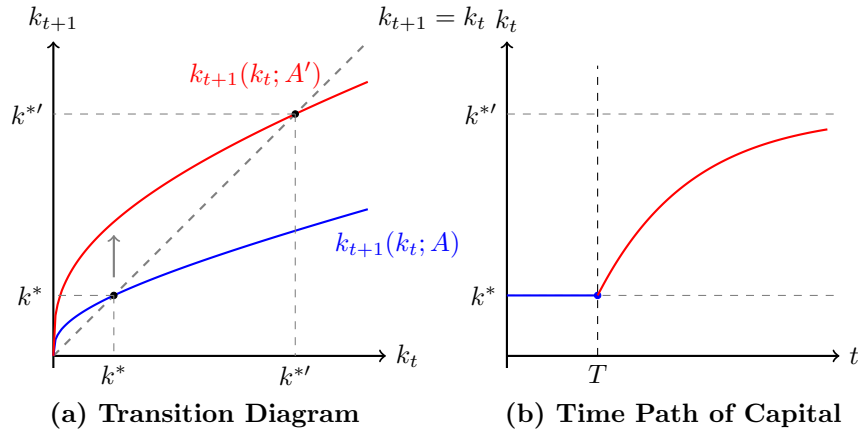
is exceptionally high, meaning investment vastly outpaces depreciation. As capital accumulates, the marginal returns diminish, causing the growth rate to gradually decelerate until the economy lands softly at the steady state.

## 5.3 When Variables Change

### 5.3.1 Technology Progress

Suppose the economy has already reached its initial steady state  $k^*$  with a technology level  $A$ . At a specific time  $T$ , technology unexpectedly and permanently improves to  $A' > A$ .

Because capital per worker  $k_t$  is a *state variable* (evolution is durable), it cannot jump instantly at time  $T$ . However, the technological progress causes the economy's transition curve  $k_{t+1}(k_t) = (1 - \delta)k_t + sAf(k_t)$  to shift up vertically. Consequently, starting from time  $T$ , the economy begins to grow rapidly. Due to the pronounced diminishing returns, this “catch-up growth” slows down quickly, and the capital stock  $k_t$  transitions smoothly yet sharply towards the new, higher steady state  $k^{*'}$ .

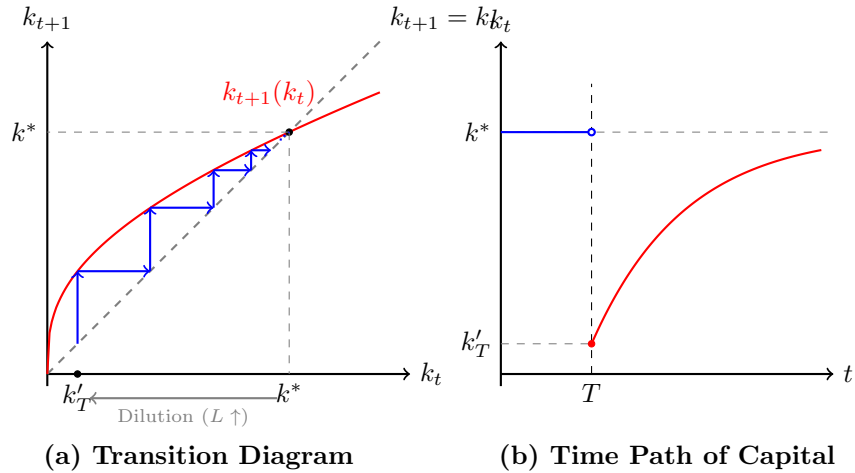


### 5.3.2 One-Time Population Shock

Suppose the economy has already reached its steady state  $k^*$  with a given technology level  $A$  and a constant population  $L$ . At a specific time  $T$ , the population exogenously and instantly increases to  $L' > L$  (e.g., due to a sudden migration event).

Unlike a change in technology or saving rates, a one-time change in the level of population does not shift the transition curve  $k_{t+1}(k_t)$ . However, it directly impacts the per-worker state variable. Because the total capital stock  $K_T$  is durable and fixed at time  $T$ , the sudden increase in the denominator  $L$  causes the capital per worker  $k_T$  to jump down instantly to a lower level,  $k'_T = K_T/L' < k^*$ .

At this lower capital level, investment per worker now exceeds depreciation per worker. Consequently, starting from time  $T$ , the economy experiences temporarily high “catch-up growth,” accumulating capital per worker until it eventually converges back to the original steady state  $k^*$ . Take a special note that the total output  $Y$  will be higher in the new steady state (due to more  $L$ ), but output per worker  $y^*$  remains unchanged.



### 5.4 Non-Autonomous Dynamics and the Balanced Growth Path

What happens if technology grows exponentially over time, such that  $A_t = A_0 e^{g_A \cdot t}$ ?

Now that the exogenous sequence  $\{A_t\}_{t=0}^\infty$  is known and strictly increasing, the Solow model becomes a *non-autonomous* dynamic system.<sup>2</sup> We can track the sequence of capital  $\{k_t\}_{t=0}^\infty$  by iterating the law of motion for  $k_{t+1}$ :

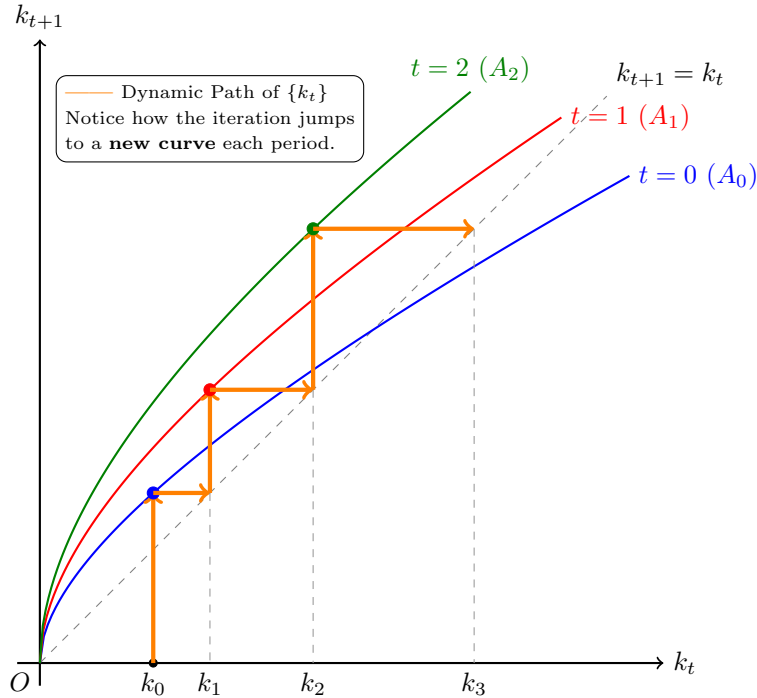
$$\begin{aligned} k_1 &= (1 - \delta)k_0 + sA_0k_0^\alpha \\ k_2 &= (1 - \delta)k_1 + sA_1k_1^\alpha \\ k_3 &= (1 - \delta)k_2 + sA_2k_2^\alpha \\ &\vdots \end{aligned}$$

Geometrically, instead of a single curve, we plot a *family of curves* on the  $(k_t, k_{t+1})$  phase plane:

$$k_{t+1} = (1 - \delta)k_t + sA_t k_t^\alpha$$

Because  $A_t$  is strictly increasing with  $t$ , the transition curve shifts upward every period.

<sup>2</sup>In the theory of dynamical systems, a system is called *autonomous* if its law of motion does not explicitly depend on time (i.e.,  $x_{t+1} = f(x_t)$ ). The baseline Solow model without technological progress is autonomous, yielding a single, fixed transition curve. When  $A_t$  grows over time, the transition equation becomes  $k_{t+1} = f(k_t, t)$ . This explicit dependence on  $t$  makes the system *non-autonomous*, which graphically manifests as the transition curve shifting in every period.



Since the transition curve is continuously shifting upward, a static steady state (where  $k_{t+1} = k_t$ ) no longer exists. Instead, we look for a *Balanced Growth Path*: a trajectory where key macroeconomic variables (like  $k_t$  and  $y_t$ ) grow at constant, time-invariant rates from the very first period.

### Claim: Existence of the Balanced Growth Path

Given a constant growth rate of technology  $g_A$ , there exists a specific initial capital stock  $k_0^*$  and a constant growth rate of capital  $g_k$ , such that the sequence:

$$k_t = k_0^* e^{g_k \cdot t}$$

is an exact solution to the dynamic model for all  $t \geq 0$ .

#### *Proof for Claim.*

We construct the proof using the standard *Guess and Verify* method.

- **Step 1: Guess**

Suppose the claim is true, meaning  $k_t$  grows at a constant rate  $g_k$ . We substitute  $k_t = k_0 e^{g_k t}$  and  $k_{t+1} = k_0 e^{g_k(t+1)}$  into the law of motion:

$$k_0 e^{g_k(t+1)} = (1 - \delta) k_0 e^{g_k t} + s A_0 e^{g_A t} (k_0 e^{g_k t})^\alpha$$

- **Step 2: Verify by matching coefficients**

Divide both sides by the term  $k_0 e^{g_k t}$ :

$$e^{g_k} = (1 - \delta) + s A_0 k_0^{\alpha-1} e^{g_A t} e^{\alpha g_k t} e^{-g_k t}$$

Rearranging the terms yields:

$$e^{g_k} + \delta - 1 = sA_0k_0^{\alpha-1}e^{[g_A+(\alpha-1)g_k]t}$$

For this equation to hold true for *all*  $t$  (which is the definition of balanced growth), the right-hand side must be independent of time  $t$ . Therefore, the exponent on  $e$  must be exactly zero:

$$g_A + (\alpha - 1)g_k = 0 \implies g_k = \frac{g_A}{1 - \alpha}$$

• **Step 3: Pin down the initial condition**  $k_0^*$

Now that the exponential term is  $e^0 = 1$ , we can solve for the unique initial capital  $k_0^*$  that places the economy immediately on the BGP:

$$k_0^* = \left( \frac{sA_0}{e^{g_k} + \delta - 1} \right)^{\frac{1}{1-\alpha}}$$

As long as the economy starts at this specific  $k_0^*$ , capital will grow forever at the constant rate  $g_k = g_A/(1 - \alpha)$ .

## 5.5 The Lucas Critique

### Lucas Critique

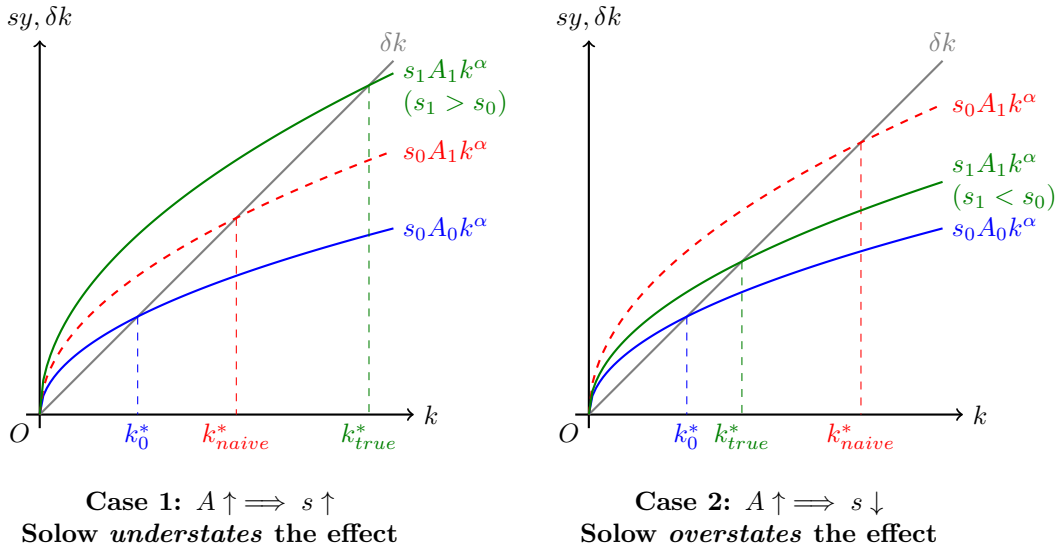
Assuming structural parameters remain invariant to policy or environmental changes leads to systematic forecasting errors.

One of the most fundamental criticisms of the baseline Solow model is its assumption that the saving rate  $s$  is an exogenous constant. In reality, saving is an optimal choice made by forward-looking households. If the economic environment changes, rational agents will adjust their behavior.

Consider a positive technological shock ( $A \uparrow$ ). In the standard Solow model, the investment curve mechanically shifts up, determining a new “naive” steady state. However, if the technological improvement alters the return to capital or lifetime income, the household’s optimal saving rate  $s$  will also change.

- **Understating the Effect:** If an increase in  $A$  incentivizes households to save more ( $s \uparrow$ ), the investment curve shifts up twice: once due to  $A$ , and again due to  $s$ . The naive Solow model will *understate* the true increase in steady-state capital.
- **Overstating the Effect:** Conversely, if higher  $A$  leads to a wealth effect that causes households to consume more and save less ( $s \downarrow$ ), the secondary downward shift in  $s$  partially offsets the initial shock. The naive Solow model will *overstate* the true impact.

The true profundity of the Lucas Critique extends far beyond a simple complaint about the Solow model’s saving rate. It sparked a methodological revolution that forced modern macroeconomics to develop rigorous *micro-foundations*.



Prior to ?, macro-econometric models relied heavily on historical relationships between aggregate variables (treating variables like  $s$  or the marginal propensity to consume as structural constants). Lucas argued that these are merely *reduced-form* outcomes of underlying optimization problems. When the government changes an economic policy or when technology shifts, the constraints of the optimization problem change, and rational agents will immediately alter their behavior. Consequently, historical data becomes useless for predicting the effects of new policies.

To avoid the Lucas Critique, macroeconomic models must be built upon *deep parameters* (or structural parameters) that are genuinely invariant to policy changes. These include:

- **Preferences:** Time discount factor ( $\beta$ ), coefficient of relative risk aversion ( $\gamma$ ).
- **Technology:** Capital share ( $\alpha$ ), depreciation rate ( $\delta$ ).

This critique naturally paves the way for the next evolution in our theory: the *Neoclassical Growth Model* (often formalized as the *Ramsey-Cass-Koopmans model*). Instead of assuming an exogenous saving rule, this framework models the economy as a dynamic general equilibrium where households and firms actively optimize. In such a micro-founded setting, the saving behavior organically emerges as an endogenous outcome, perfectly immune to the Lucas Critique.

**Remark (Chapter Summary).**

- **No long-run growth from capital alone.** Diminishing returns to capital pin down a unique steady state  $k^*$ . Without exogenous TFP growth, capital accumulation eventually stops driving output growth.
- **Catch-up growth and conditional convergence.** An economy below its steady state grows faster than one near it. Cross-country differences in  $A, \delta, s$  explain why countries converge to different  $k^*$ .
- **Balanced Growth Path.** With  $A_t = A_0 e^{g_A t}$ , the model admits a BGP solution where  $k_t, y_t$  grow at constant rate  $g_A / (1 - \alpha)$ . Capital deepening still happens, but

technology is the long-run engine.

- **The Lucas Critique.** The exogenous saving rate  $s$  is a behavioral residual, not a structural parameter. Policy or technology changes that shift  $s$  are not captured in the Solow framework. This motivates the next chapter's optimization-based formulation.
- **The model as a launching pad.** The Solow setup is the basic accounting structure that all subsequent growth-and-cycles models extend, from the Ramsey–Cass–Koopmans neoclassical model (Chapter 6) through RBC and beyond.

## Part III

# Problem Sets and Solutions

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