

# 2015 Midterm Exam, Advanced Econometrics

Nov. 12, 2015

You have 120 minutes to finish all the five questions.

1. Suppose we want to estimate the following equation using 4 observations:

$$y = \alpha + \beta x + \varepsilon,$$

Obs.	y	x
1	-3	1
2	3	2
3	1	3
4	1	4

You may need the following information when answering the above questions:

$$t_{95, 4} = 2.13, t_{97.5, 4} = 2.78, t_{95, 3} = 2.35, t_{97.5, 3} = 3.18, t_{95, 2} = 2.92, t_{97.5, 2} = 4.30.$$

- (10 points) Calculate  $\hat{\alpha}$  and  $\hat{\beta}$ .
  - (5 points) Calculate the variance-covariance matrix for  $\hat{\alpha}$  and  $\hat{\beta}$ .
  - (5 points) Calculate  $R^2$  and adjusted  $R^2$ .
  - (5 points) Construct the 95% confidence interval for  $\alpha$  and  $\beta$ .
  - (5 points) Construct the prediction interval at 95% confidence level for  $x=2$ .
2. For the regression model  $y_i = \alpha + \beta x_i + \varepsilon_i$ ,
- (5 points) Show that the least squares equations imply  $\sum_i e_i = 0$  and  $\sum_i x_i e_i = 0$ .
  - (5 points) Show that the solution for the constant term is  $a = \bar{y} - b\bar{x}$ .
  - (5 points) Show that the solution for  $b$  is  $b = [\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})] / [\sum_{i=1}^n (x_i - \bar{x})^2]$ .
3. (15 points) In the regression  $y = X\beta + \varepsilon$  with restriction  $R\beta = r$ . Let  $b$  be the OLS estimator of  $\beta$  without restriction, and  $b_*$  be the OLS estimator with restriction. Prove that  $Var[b_*|X] \leq Var[b|X]$  whether or not the restriction is true.

4. (10 points) A multiple regression of  $y$  on a constant,  $x_1$  and  $x_2$  produces the following results:  $\hat{y} = 4 + 0.4x_1 + 0.9x_2$ ,  $R^2 = 8/60$ ,  $e'e = 520$ ,  $n = 29$ ,

$$X'X = \begin{bmatrix} 29 & 0 & 0 \\ 0 & 50 & 10 \\ 0 & 10 & 80 \end{bmatrix},$$

and you may (though probably not) need the following matrix

$$(X'X)^{-1} = \begin{bmatrix} 0.035 & 0 & 0 \\ 0 & 0.021 & -0.003 \\ 0 & -0.003 & 0.013 \end{bmatrix}.$$

Test the hypothesis that the slope on  $x_1$  is 0 by running the restricted regression and comparing the two sums of squared deviations (Hint:  $F[J, n - K] = \frac{(e'_*e_* - e'e)/J}{e'e/(n-K)}$ , and when  $\alpha = 0.05$ ,  $F[1, 26]=4.23$ ).

5. Consider the linear model with two independent variables  $y = \alpha x + \beta z + u$ . Assume that  $x$  and  $z$  are exogenous (nonrandom),  $u$  has zero mean and  $E[uu'|x, z] = \sigma^2 I$ . You are asked to investigate the properties of three alternative estimators for  $\beta$ .
- Let  $b$  be the estimate resulting from a least-squares regression of  $y$  on  $x$  and  $z$ .
  - Let  $b^*$  be the estimate obtained by regressing  $y$  on  $z$  alone.
  - Let  $b^{**}$  be the estimate resulting from the following step-wise procedure: first one regresses  $y$  on  $x$  alone, obtaining the estimate  $a^*$ ; then one regresses the residual vector  $y - a^*x$  on  $z$ , obtaining  $b^{**}$ .
- (a) (10 points) Derive expressions for the mean and for the variance of each of the three estimators.
- (b) (5 points) Show that the expected value of  $b^*$  need not have the same sign as  $\beta$ , but that the expected value of  $b^{**}$  does have the correct sign.
- (c) (5 points) Show that, conditional on the regressors,  $Var[b^{**}] \leq Var[b^*] \leq Var[b]$ .