Please write your name here: *This exam has six pages.*

Econ 501 Final — 2025 — Joris Pinkse

You are allowed one piece of letter size paper with writing on one side. Show your work throughout. Questions are presented in (what I think is an) increasing order of difficulty/work.

1. Suppose that $\mathbf{x}$ is continuously distributed with distribution function $F_{\mathbf{x}}$. Derive the density function of $\mathbf{y} = F_{\mathbf{x}}(\mathbf{x})$.

2. Suppose that $\mathbf{x}, \mathbf{y}$ are independent standard exponential, i.e. their density function is $\exp(-t)\mathbb{1}(t > 0)$. Derive the density function of $\mathbf{x} + \mathbf{y}$.

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3. Let $\{\mathbf{x}_i\}$ be i.i.d. draws from a $U(0,1)$ distribution. Determine the limit distribution of $n(1 - \max_{i=1,\ldots n} \mathbf{x}_i)$.

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4. Suppose that $\{\mathbf{x}_i\}$ is an i.i.d. sequence with distribution function $F(x) = \exp[-\exp(-x)]$. Determine $\mathbb{E} \max_{i=1,\ldots,n} \mathbf{x}_i$. *Hint:* $\gamma = -\int_0^\infty \exp(-x)\log(x)\,\mathrm{d}x$ *is the Euler-Mascheroni constant, which cannot be simplified further.*

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5. Suppose $\{\mathbf{x}_i\}$ is i.i.d. and continuous with support $[1, \infty)$. Prove that $\mathbb{E}(1/\bar{\mathbf{x}}) \to 1/\mu$ where $\bar{\mathbf{x}}$ is the sample mean and μ the expectation.

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6. Suppose that $\Omega(\theta) = \mathbb{E}\log[(\mathbf{x}_i-\theta)^2+1]$, $\hat{\Omega}(\theta) = n^{-1}\sum_{i=1}^{n}\log[(\mathbf{x}_i-\theta)^2+1]$, and that $\{\mathbf{x}_i\}$ is i.i.d. with a density function that is symmetric around the 'truth' θ_0. Suppose further that we have shown that $\hat{\boldsymbol{\theta}} = \arg\min_{\theta\in\Theta}\hat{\Omega}(\theta)$ a consistent estimator of $\theta_0 = \arg\min_{\theta\in\Theta}\Omega(\theta)$ for θ_0 in the interior of a compact set Θ.

   (a) Derive the asymptotic distribution of $\hat{\boldsymbol{\theta}}$ expressed in terms of expectations of functions of $\mathbf{y}_i = \mathbf{x}_i - \theta_0$.

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(b) Derive a $100(1-\alpha)\%$ confidence interval for θ_0.

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7. Derive the limit distribution of $n^{-3/2}\sum_{i,j=1}^{n}[\exp[-(\mathbf{x}_i-\mathbf{x}_j)^2/2]-\mu]$ with $\mu = \mathbb{E}\exp[-(\mathbf{x}_1-\mathbf{x}_2)^2/2]$ if $\{\mathbf{x}_i\}$ is i.i.d. standard normal. *You don't have to work out the final variance, working out an expression for the random variable whose variance is taken is sufficient. Hint: the standard normal density ϕ satisfies $\int \phi(x)\phi(x-y)\,\mathrm{d}x = \phi(y/\sqrt{2})/\sqrt{2}$, which is the density of a $N(0,2)$.*

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That's all folks. It has been great teaching you. Good luck with your remaining exams.