Econ 501 Midterm 2— November 18, 2025

You are allowed one piece of letter size paper with writing on one side only. Show your work throughout.

1. (a) Suppose that $\{(\mathbf{x}_i, \mathbf{y}_i)\}$ is an i.i.d. sequence of random vectors whose elements have means $\mu_x > 0,\ \mu_y \in \mathbb{R}$, and variances $\sigma_x^2 = \sigma_y^2 = 1$. Assume that $\mathbf{x}_i, \mathbf{y}_i$ are independent. Determine the convergence rate α_n of $\mathbf{z}_n = \bar{\mathbf{y}} \log \bar{\mathbf{x}} - \mu_y \log \mu_x$ and the limit distribution of $\alpha_n \mathbf{z}_n$, both for the case in which $(\mu_x, \mu_y) = (1, 0)$ and for the case in which it is not. In either case, if the limit distribution is normal then please make sure you get explicit expressions for the mean and variance.

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(b) Suppose that $\{\mathbf{x}_i\}$ is i.i.d. with an even density (i.e. it is symmetric around zero) and $\mathbb{E}\sqrt{|\mathbf{x}_i|} < \infty$. What is the best rate α_n that you can find for which $\bar{\mathbf{x}} = O_p(\alpha_n)$? A better rate yields more points.

2. True or false? Explain briefly. No points without a correct explanation.

(a) A parametric estimator $\hat{\theta}$ of θ_0 is typically such that $\hat{\theta} - \theta_0 = O_p(n^{-1/2})$.

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(b) $\mathbf{x}_n = O_p(n^{1/2}) \Rightarrow \mathbf{x}_n = o_p(1)$.

3. Suppose that $\{(\mathbf{x}_i, \mathbf{y}_i)\}$ is i.i.d. and such that $\mathbf{y}_i \mid \mathbf{x}_i \sim E(1/\mathbf{x}_i)$ with $\mathbb{E}\mathbf{x}_i = 0$ and $\mathbb{V}\mathbf{x}_i = 1$. Derive the asymptotic variance of $\sqrt{n}(\bar{\mathbf{y}} - \mu_y)$. *Hint: an exponential distribution with parameter λ has density function $\lambda \exp(-\lambda x)\mathbb{I}(x > 0)$, mean $1/\lambda$, and variance $1/\lambda^2$.*

4. Suppose that $\bar{\mathbf{x}}$ is the sample mean of an i.i.d. mean zero, (positive) finite variance, sequence. Rank the following objects in terms of their rate of convergence (fastest first): (a) $\bar{\mathbf{x}}$; (b) $\bar{\mathbf{x}}^2$; (c) $\exp(-\bar{\mathbf{x}}) - 1$; (d) $\exp(-\bar{\mathbf{x}}^2) - 1$; (e) $\exp(-1/\bar{\mathbf{x}}^2)$; (f) $\mathbb{1}(|\bar{\mathbf{x}}| \geq \epsilon)$ for fixed $0 < \epsilon < \infty$. Indicate if multiple choices converge equally fast.

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5. (a) Suppose that $\theta_0 = \arg\min_{\theta\in\Theta} \Omega(\theta)$ where $\Theta = [0,2]$ and $\Omega(\theta) = \mathbb{E}\big(|\mathbf{x}_i - \theta| - |\mathbf{x}_i|\big)$ with $\mathbf{x}_i \in \mathbb{R}$ has a continuous density function and $\{\mathbf{x}_i\}$ i.i.d.. Establish consistency of $\hat{\theta} = \arg\min_{\theta\in\Theta} n^{-1} \sum_i \big(|\mathbf{x}_i - \theta| - |\mathbf{x}_i|\big)$ for θ_0.

(b) Explain why if the distribution of $\mathbf{x}_i$ were not continuous then consistency of $\hat{\theta}$ may not obtain.

Please write your name here: *This exam has six pages.*

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