Econ 521	Vijay Krishna
Final Exam	Spring 2025

Instructions *Please answer all 3 questions. Make sure that you provide reasoning for your answers. Use the paper supplied as answer sheets, put your name on each page and number them. Do not staple your answers.*

1. Consider a two-sided matching problem between students in the set $S = \{s_1, s_2, s_3\}$ and colleges in the set $C = \{c_1, c_2, c_3\}$. Each college has only one slot. The (strict) preferences of the partiess are as follows:

$P(s_1)$	$P(s_2)$	$P(s_3)$		$P(c_1)$	$P(c_2)$	$P(c_3)$
c_1	c_1	c_1		s_1	s_1	s_1
c_2	c_2	c_3		s_2	s_3	s_2
c_3	$\emptyset$	c_2		s_3	s_2	$\emptyset$

where $\emptyset$ denotes that all other matches are deemed unacceptable. Thus, s_2 deems c_3 to be unacceptable and c_3 deems s_3 to be unacceptable

(a) Find the stable matching from the Gale-Shapley algorithm when students make offers (apply).

(b) Find the corresponding matching when colleges make admission offers.

(c) Are there any other stable matchings?

2. Consider the game G below:

	H	M	L
H	3, 3	1, 1	0, 5
M	1, 1	2, 2	0, 1
L	5, 0	1, 0	−1, −1

and let $G_\delta(\infty)$ denote the game where G is infinitely repeated and both players use a common discount factor $\delta \in (0, 1)$ to evaluate future payoffs. There is perfect monitoring—that is, all past choices are observed by both players.

(a) Consider the following outcome paths:

(i) (H, H), (H, H), ...
(ii) (M, M), (M, M), ...

Suppose the players play the "trigger" strategy: start with (i) and continue if there are no deviations. If there is any deviation from (i) then play (ii) forever.

For what values of δ do the trigger strategies constitute a subgame perfect equilibrium of $G_\delta(\infty)$?

(b) Now consider the following outcome paths:

(i) (H, H), (H, H), ...
(ii) (L, L), (H, H), (H, H), ...

Suppose the players play the "forgiving" strategy: start with (i) and continue if there are no deviations. If there is any deviation from (i) start (ii). If there is any deviation from (ii), then restart (ii).

For what values of δ do the forgiving strategies constitute a subgame perfect equilibrium?

3. Consider the game depicted in Figure 1. Player 1 can be of two types: strong (S) with probability p and weak with probability $1-p$. Knowing his own type, player 1 takes one of two actions; he can give a gift to player 2 (G) or not (N). If player 1 chooses not to give a gift (N), the game ends. If he gives a gift (G), player 2 takes one of two actions: accept the gift (A) or reject it (R). Player 2 does not know player 1's type. The resulting payoffs are as specified.

   (a) Show that the game has a pooling perfect Bayesian equilibrium (PBE) in which both types of player 1 choose N. Carefully specify the strategies and beliefs. (*Note*: In a pooling equilibrium, different types take the same action.)

   (b) Show that the game has a pooling PBE in which both types of player 1 choose G if and only if $p \geq \frac{1}{2}$.

   (c) Does the game have a PBE which is *separating*? Does it have a Nash equilibrium which is separating? (*Note*: In a separating equilibrium, different types take different actions.)

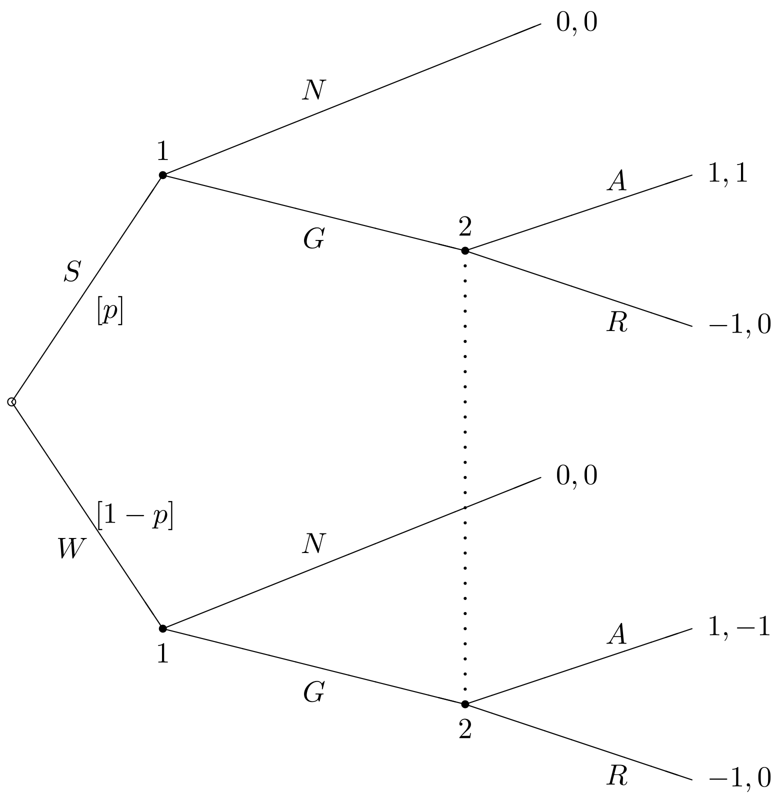

Figure 1: