Econ 521 — Vijay Krishna

First Mid-Term Exam — February 18, 2025

Instructions *Please answer all four questions. Make sure that you provide reasoning for your answers.*

1. Find a Nash equilibrium (pure or mixed) of the following incompletely specified game (where "$\cdot$" denotes an unknown payoff):

	L	C	R
U	$\cdot$, 1	3, 2	2, 0
M	$\cdot$, $\cdot$	0, $\cdot$	3, $\cdot$
D	$\cdot$, 3	5, 0	1, 6

2. Consider the following game G:

	A	B
A	3, 2	1, 1
B	4, 3	2, 4

   (a) Find all pure strategy Nash equilibria of G.

   (b) Now consider a game Γ with the same payoffs as above but suppose that the players move sequentially. Player 1 (who chooses rows) moves first, and player 2 (who chooses columns) moves after perfectly observing the choice of player 1. Find all subgame perfect equilibria of the sequential game Γ and compare your answer to that in part (a).

   (c) Now consider a game Γ_ε in which player 1 moves first, but her moves are only *imperfectly* observed by player 2. When player 1 chooses A, player 2 receives a "signal" a with probability $1-\varepsilon$ and a signal b with probability ε, where $0 < \varepsilon < \frac{1}{4}$. Likewise, when player 1 chooses B, player 2 receives a signal a with probability ε and a signal b with probability $1-\varepsilon$.

      1. Draw the extensive form associated with this game.
      2. Show that there is a Nash equilibrium of the game with imperfectly observed actions in which both players choose B.

3. Let $G = (S_i, u_i)_{i=1}^{i=2}$ be a finite two-player game given in strategic form. Define Γ to be an extensive form game of perfect information where first player 1 chooses $s_1 \in S_1$. Player 2 is informed of player 1's choice and then chooses $s_2 \in S_2$. For any pair (s_1, s_2), the payoffs in Γ are the same as in G.

   (a) Show that: For every *pure strategy* Nash equilibrium of G there is a *pure strategy* Nash equilibrium of Γ in which, in equilibrium, the players make the same choices as in the equilibrium of G.

   (b) Construct an example to show that the statement in part (a) does not hold for *mixed strategy* equilibria.

   (c) Construct an example to show that the statement in part (a) does not hold for *pure strategy subgame perfect* equilibria.

Question #4 is overleaf.

4. Consider the following model of an arms race between two countries. Each country i selects a level of "military capability" $x_i \in [0,1]$. The payoff to country i when the two countries choose capabilities (x_1, x_2) is

$$u_i(x_1, x_2) = g(x_i - x_j) - c(x_i) \qquad (j \neq i)$$

where $g : [-1,1] \to \mathbb{R}$ is a strictly increasing ($g' > 0$) and strictly concave function ($g'' < 0$) and $c : [0,1] \to \mathbb{R}$ is a strictly increasing ($c' > 0$) and strictly convex function ($c'' > 0$). The function g represents the "gain" to a country from having a military advantage over the other and the function c represents the cost of acquiring military capability.

(a) Show that the game described is supermodular

(b) Show that the game has a *symmetric pure strategy* equilibrium (x^*, x^*).

(c) Can this game have multiple symmetric pure strategy equilibria?

(d) Can this game have asymmetric pure strategy equilibria (x_1^*, x_2^*), where $x_1^* \neq x_2^*$?