Econ 521 Vijay Krishna

Second Mid-Term Exam April 3, 2025

Instructions *Please answer all three questions. Be sure to provide reasoning for your answers.*

1. Carefully define the following terms.

   (a) Nash's axiom of *independence of irrelevant alternatives* (IIA) for bargaining problems.

   (b) The *revenue equivalence principle* of auctions.

   (c) A *stable* match in a situation in which there is one-to-one matching between students and colleges.

2. Consider the Rubinstein infinite-horizon alternating offer bargaining model but with the following preferences. If an agreement (z, t) is reached (a split of $(z, 1-z)$ is agreed upon at time t), then the payoff to player 1 is $z - (t-1)\,c_1$ and the payoff to player 2 is $(1-z) - (t-1)\,c_2$ where c_i is player i's constant per-period cost of delay and satisfies $0 < c_i < 1$. Note that there is no discounting of future gains; rather there is constant per-period cost of delay. As usual, let G_i denote the game in which player i makes the first offer.

   (a) First, suppose that $c_1 = c_2 = c$. Show that if $x \in [0,1]$ and $y \in [0,1]$ are such that $x = y + c$, then there is a subgame perfect equilibrium of G_1 in which a split of $(x, 1-x)$ is achieved with no delay and a subgame perfect equilibrium of G_2 in which a split of $(y, 1-y)$ is achieved, again with no delay. Write down the equilibrium strategies carefully.

   (b) Now, suppose that $c_1 < c_2$. Show that there is a subgame perfect equilibrium of G_1 in which a split of $(1, 0)$ is achieved without delay and a subgame perfect equilibrium of G_2 in which a split of $(1 - c_1, c_1)$ is achieved without delay. Again, write down the equilibrium strategies carefully.

3. There is a single object for sale and two (2) potential buyers are bidding for the object. Bidder i assigns a value of X_i to the object. Each X_i is a random variable that is independently and *uniformly* distributed on the interval $[0,1]$. Bidder i knows the realization x_i of X_i and only that other bidder's values are independently and uniformly distributed on $[0,1]$. The object being sold has a use value of 0 to the seller.

   (a) Suppose that the seller decides to use the following "hybrid" auction—one that is a mixture of a first-price and a second-price auction. Each bidder i submits a sealed bid b_i. If $b_i > b_j$, then bidder i wins the object and pays $\alpha b_i + (1-\alpha)\, b_j$ where α is a fixed and known parameter satisfying $0 \leq \alpha \leq 1$. Show that it is an equilibrium for each bidder to follow the strategy:

   $$\beta(x) = \frac{x}{1+\alpha}$$

   (b) Does this "hybrid" auction satisfy the assumptions of the revenue equivalence principle?

   (c) Can the seller benefit by setting a non-zero reserve price in the hybrid auction? What is the optimal reserve price?