Econ 521 — Vijay Krishna

Final Exam — Spring 2026

Instructions *Please answer all 3 questions. Make sure that you provide reasoning for your answers. Use the paper supplied as answer sheets, put your name on each page and number them.*

1. Consider a two-sided matching problem between students in the set $S = \{s_1, s_2, s_3\}$ and colleges in the set $C = \{c_1, c_2, c_3\}$. Each college has *only one slot*. The (strict) preferences of the agents (top is best etc.) are:

$P(s_1)$	$P(s_2)$	$P(s_3)$	$P(c_1)$	$P(c_2)$	$P(c_3)$
c_1	c_2	c_1	s_2	s_1	s_1
c_2	c_1	c_2	s_1	s_2	s_2
c_3	c_3	c_3	s_3	s_3	s_3

   (a) Suppose that matching is determined by the Gale-Shapley algorithm when students make offers (apply). What is the resulting stable matching μ_S?

   (b) Is the matching $\mu = \{(c_1, s_3), (c_2, s_1), (c_3, s_2)\}$ stable?

   (c) Show that college c_1 can manipulate the outcome in its favor by submitting the false preference $P'(c_1) = s_2 \succ s_3 \succ s_1$ instead of $P(c_1)$. In other words, if μ'_S denotes the outcome of the Gale-Shapley algorithm with students proposing when c_1 submits the false preference $P'(c_1)$ (and all others submit their true preferences), then c_1 is better-off in the matching μ'_S rather than μ_S.

2. Consider the prisoners' dilemma game G:

	C	D
C	2, 2	−1, 3
D	3, −1	0, 0

   and denote by $G(2)$ its two-fold repetition in which before playing in the second period, both players observe all choices in period 1.

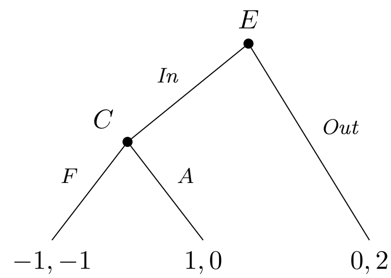
   (a) Show that the unique *Nash equilibrium outcome* of $G(2)$ is $(D, D), (D, D)$. (Note that the question is not about subgame perfect equilibria but rather Nash equilibria.)

   (b) Next consider the modified prisoner's dilemma game G':

	C	D	E
C	2, 2	−1, 3	−2, −2
D	3, −1	0, 0	−1, −2
E	−2, −2	−2, −1	−2, −2

   Show that $(C, C), (D, D)$ is a Nash equilibrium outcome of $G'(2)$. Is it a subgame perfect equilibrium outcome of $G'(2)$ as well?

3.



Consider the entry game above in which a chain-store (player C) faces a threat of entry from an entrant (player E). E can either enter (play *In*) or not (play *Out*) and C can either fight entry (play F) or accommodate it (play A). The payoffs are given in the tree where the first number is the entrant's payoff and the second is the chain-store's payoff.

Suppose that there are two (2) periods and in each period, the game above is played against a *different* entrant. In period 1, the chain-store faces the first entrant, E_1, and in period 2, it faces E_2. Outcomes in period 1 are observed by both C and E_2.

But there is a probability $\varepsilon < \frac{1}{4}$ that the chain-store's payoffs are not as specified above but rather such that the chain-store prefers to fight (play F) rather than to accommodate (play A). Call such a chain-store "irrational".

Consider the following strategies in the two-period game when the probability that the chain-store is irrational is $\varepsilon < \frac{1}{4}$.

(i) In period 1, the entrant E_1 enters for sure. The chain-store fights with probability $\varepsilon/(1-\varepsilon)$.

(ii) In period 2, the entrant E_2 enters for sure if the outcome in period 1 was (In, A) and enters with probability $\frac{1}{2}$ if the outcome in period 1 was (In, F).

(a) Show that the strategies in (i) and (ii) constitute a perfect Bayesian equilibrium (PBE). Make sure you specify players' beliefs.

(b) (**Optional for extra credit**) Show that the outcome in (a) is the unique PBE outcome of this game.