Econ 521 | Vijay Krishna

Second Mid-Term Exam | March 31, 2026

Instructions *Please answer all three questions. Be sure to provide reasoning for your answers.*

1. Carefully define the following terms.

   (a) A *bargaining solution* (in the axiomatic bargaining framework)

   (b) The *virtual value function* associated with a distribution F.

   (c) The *Vickrey-Clarke-Groves (VCG) mechanism.*

2. Two players bargain via the Rubinstein alternating offer procedure to divide a "pie" of size 1.

   (a) Suppose that the players have different utility utility functions: $u_1(z) = z$ and $u_2(z) = \sqrt{z}$ and use a common discount factor $\delta \in (0, 1)$ to discount future utilities. How will the pie be split in the unique subgame perfect equilibrium?

   (b) Now suppose that both players have identical linear utility functions $u(z) = z$ but use different discount factors $\delta_1 < \delta_2$ (both in $(0, 1)$) to discount future utilities.

      1. How will the pie be split?
      2. For what value of δ_2 is the split in part (b) the same as in part (a) when the common discount factor $\delta = \delta_1$?

3. There is a single object for sale and two (2) potential buyers are bidding for the object. Bidder i assigns a value of X_i to the object. Each X_i is independently and *uniformly* distributed on the interval $[0, 1]$. Bidder i knows the realization x_i of X_i and only that other bidder's values are independently and uniformly distributed on $[0, 1]$. The object being sold has a use value of 0 to the seller.

   (a) Suppose that the seller sells the object using a *second-price* auction (SPA) and sets a reserve price of $r > 0$. What is the optimal reserve price r^{SPA} that a seller should set in a second-price auction?

   (b) Now suppose that the seller sells the object using a *first-price* auction (FPA) and sets a reserve price of $r > 0$. What is the equilibrium bidding strategy $\beta_r : [0, 1] \to \mathbb{R}$ when $r > 0$? What is the optimal reserve price r^{FPA} in a first-price auction? How does r^{FPA} compare to r^{SPA}?

   (c) Finally, suppose that the seller sells the object via a second-price auction but instead of setting a reserve price $r > 0$, the seller charges each bidder an entry fee $c > 0$. Thus, in order to participate in the auction each bidder must pay a non-refundable amount c to the seller prior to bidding. What is the optimal entry fee c^{SPA}?